A Convenient Mesh Rotation Method of Finite Element Analysis Using Sub-matrix Transformation Approach

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Abstract — This paper presents a novel sub-matrix transformation method on mesh rotation problems in the finite element analysis (FEA) of electric machines. This proposed approach is simple, convenient and readily implementable. For each rotor position, only the transformation matrix which has fixed regular pattern versus the rotor position displacement needs to be modified. Transformation matrices with first-order element, second-order element as well as with periodic and anti-periodic boundary conditions have been developed. By using sub-matrix transformation, the mesh coupling can be realized automatically, efficiently and with minimal computing burden, since all the coefficients of the FEA system equation are stored in the sub-matrixes and they do not need to be re-assembled for different rotor positions. Formulae derivation and theoretical analysis are presented together simulation results to verify the validity of proposed method.

I. INTRODUCTION

Finite element analysis (FEA) has been successfully and widely used in the modeling of electric machines for decades [1]. For rotary electric machines, it is necessary to take into account the relative angular movements between stator and rotor. As the rotor moves, the nodes on the sliding surface of the stationary and moving parts may not necessarily overlap with each other. Many approaches have been proposed to deal with such mesh connection problems, including moving band approach, boundary element method, Lagrange multipliers method, sliding-surface approach. Each of them has its merits and demerits. For the moving band technique, an automatic remeshing process in the airgap for each rotor position can be used to join the stationary and moving parts together. However, unavoidable changes in mesh topology may result in numerically noisy solutions in transient computation [2]. Boundary element method can resolve the mesh coupling problem efficiently with less unknown variables than other approaches. Nonetheless, the main demerit is that it leads to a fully populated matrix, which takes up more computing burden and both storage and computational time will grow [3]. Lagrange multiplier method needs the establishment of specific formulation for each model and is not convenient and hence time consuming in numerical computation [4]. As an alternative, sliding surface approach based on lock-step method or interpolation method is used for accurate and efficient modeling of rotation [5]. The disadvantage is that the implementation is complex.

In this paper, a sub-matrix transformation method is presented to deal with rotation problems in FEA. By using the sub-matrix transformation on the slave and master nodes as will be defined later, the magnetic field of the stationary part and the moving one can be directly coupled together. Mesh connection in each rotor position is then realized. The merit of this method is that this proposed approach is very simple, convenient and readily implementable. For each rotor position, one needs to modify only the transformation matrix which has a fixed regular pattern against the rotor position and it can be pre-determined. By using sub-matrix transformation, the mesh rotation can be realized automatically. All the coefficients of the FEA system equation are stored in sub-matrixes which do not need to be assembled again for different rotor positions. Formulae derivation and theoretical analysis are presented and simulation results are used to validate the proposed method.

II. MESH COUPLING METHOD

A. Basic System Equations

In the modeling of electric machines, the solution domain is divided into two regions along the mid airgap. The stator region (Region *I*) includes the stator and part of the airgap and is meshed independently. The rotor region (Region *II*) includes the rotor and another part of the airgap and is also meshed independently.

Using the Galerkin method to discretize the magnetic field equations, the algebraic equation in the stator and rotor can be written in sub-matrix matrix format as:

$$\begin{bmatrix} \mathbf{C}_I & \mathbf{C}_{IM} \\ \mathbf{C}_{MI} & \mathbf{C}_M \end{bmatrix} \begin{Bmatrix} \mathbf{A}_I \\ \mathbf{A}_M \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}_I \\ \mathbf{P}_M \end{Bmatrix} \tag{1}$$

$$\begin{bmatrix} \mathbf{C}_{II} & \mathbf{C}_{IS} \\ \mathbf{C}_{SI} & \mathbf{C}_S \end{bmatrix} \begin{Bmatrix} \mathbf{A}_{II} \\ \mathbf{A}_S \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}_{II} \\ \mathbf{P}_S \end{Bmatrix} \tag{2}$$

where $\boldsymbol{A}_M$ and $\boldsymbol{A}_S$ are the magnetic potentials on the master and slave nodes of the sliding surface, respectively; $\boldsymbol{A}_I$ is the magnetic potential on the nodes inside the Region *I*; *C* is the coefficient matrix; *P* is the right hand side associated with excitations.

$\boldsymbol{A}_M$ and $\boldsymbol{A}_S$ are linked together by a transformation matrix *M* and can be expressed in sub-matrix format:

$$\{\mathbf{A}_S\} = [\mathbf{M}]\{\mathbf{A}_M\} \tag{3}$$

The whole system equation is

$$\begin{bmatrix} \mathbf{C}_I & \mathbf{C}_{IM} & 0 \\ \mathbf{C}_{MI} & \mathbf{C}_M + \mathbf{M}^T\mathbf{C}_S\mathbf{M} & \mathbf{M}^T\mathbf{C}_{SII} \\ 0 & \mathbf{C}_{IIS}\mathbf{M} & \mathbf{C}_{II} \end{bmatrix} \begin{Bmatrix} \mathbf{A}_I \\ \mathbf{A}_M \\ \mathbf{A}_{II} \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}_I \\ \mathbf{P}_M + \mathbf{M}^T\mathbf{P}_S \\ \mathbf{P}_{II} \end{Bmatrix} \tag{4}$$

B. Transformation Matrix of the First-order Element

In the first-order element, the shape functions are:

$$N_k = 1 - \zeta \tag{5}$$

$$N_{k+1} = \zeta \tag{6}$$

The local coordinate is:

$$\zeta = \frac{\theta - \theta_1}{\theta_2 - \theta_1}, \qquad 0 \le \zeta \le 1 \tag{7}$$

The transformation matrix is

$$\boldsymbol{M} = N_k \boldsymbol{I}^k + N_{k+1} \boldsymbol{I}^{k+1} \tag{8}$$

C. Transformation Matrix of the Second-order Element

The master nodes on the edge of a triangle are M_k, $M_{(k+1)}$, $M_{(k+2)}$ (where $M_{(k+1)}$ is the middle point of the edge of a triangular element); the slave node on the edge of the neighbor triangle is s as shown in Fig. 1.

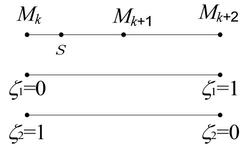

Fig. 1. The second-order line element.

A_s will be replaced by A_{Mk}, $A_{M(k+1)}$ and $A_{M(k+2)}$.

$$A_s = N_k A_{Mk} + N_{(k+1)} A_{M(k+1)} + N_{(k+2)} A_{M(k+2)} \tag{9}$$

where the shape functions are:

$$N_k = \zeta_2 (2\zeta_2 - 1) \tag{10}$$

$$N_{k+1} = 4\zeta_1 \zeta_2 \tag{11}$$

$$N_{k+2} = \zeta_1 (2\zeta_1 - 1) \tag{12}$$

where the local coordinates are:

$$\zeta_1 = \frac{\theta - \theta_0}{\theta_2 - \theta_0}, \qquad \zeta_2 = 1 - \zeta_1 \tag{13}$$

The transformation matrix is

$$\boldsymbol{M} = N_k \boldsymbol{I}^k + N_{k+1} \boldsymbol{I}^{k+1} + N_{k+2} \boldsymbol{I}^{k+2} \tag{14}$$

D. Dealing with Periodic or Anti-periodic Boundary Conditions

If the pole-pair number of electric machines is larger than one, periodic boundary conditions can be used to reduce the computing time substantially. If the solution domain only covers one pole pair, the two radial boundary edges of the solution domain have periodic boundary condition. If the solution domain covers one pole, the two radial boundary edges of the solution domain have anti-periodic boundary condition. For the periodic boundary conditions, the transform matrix has the same formulae as (8) and (14). For the anti-periodic boundary conditions, the transformation matrix for the first-order element is revised as:

$$\boldsymbol{M} = d_k \times (N_k \boldsymbol{E}^k + N_{k+1} \boldsymbol{E}^{k+1}) \tag{15}$$

where

$$\boldsymbol{E}^k = \begin{bmatrix} 0 & \cdots & 1 & & & & & \\ & 0 & & 1 & & & & \\ & & 0 & & 1 & & & \\ & & & \ddots & & \ddots & & \\ & & & & 0 & & 1 & \\ & & & & & 0 & & 1 \\ -1 & & & & & & 0 & \\ & -1 & & & & & & 0 \\ & & -1 & & & & & & 0 \end{bmatrix} \quad (\downarrow \text{column } k) \tag{16}$$

where; d_k is equal to "+1" or "-1", which is dependent on the position of the rotor. If the first slave node has periodic boundary condition with an associated master node, $d_k = +1$; if the first node has anti-periodic boundary condition with associated master node, $d_k = -1$.

The transformation matrix for the second-order element is revised as:

$$\mathbf{M} = d_k \times \left(N_k \mathbf{E}^k + N_{k+1} \mathbf{E}^{k+1} + N_{k+2} \mathbf{E}^{k+2} \right) \tag{17}$$

III. EXAMPLES AND RESULTS

In order to evaluate the performance of the proposed method, a 24-pole PM Vernier machine is designed and simulated, as shown in Fig. 2. The cogging torque is shown in Fig. 3. The design data are listed in Table I.

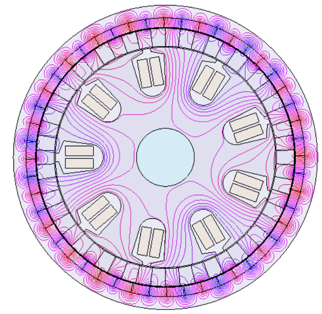

Fig. 2. Magnetic field distribution of the proposed flux modulation PM machine.

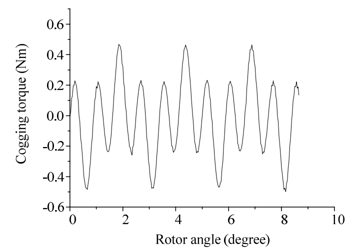

Fig. 3. Cogging torque waveforms.

TABLE I
PARAMETERS OF FLUX MODULATION PM MACHINE

Item	Flux Modulation PM Machine
Number of stator slots	9
Number of pole pairs of stator windings	3
Number of outer-PM pole pairs	24
Number of modulation poles pieces	27
Airgap length	0.6 mm
Outside diameter	184 mm
Stator diameter	133.2 mm
Modulation poles height	10.8 mm
PMs length	5.7 mm
Remanence of PMs	1.1 T
Rated speed	250 rpm
Frequency	100 Hz

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